Lösningar

1. $\bar{r}(t) = 2t^3\hat{x} + t^2\hat{y} + 5\hat{z}$ m

a. Partikelns läge efter 2s, $\bar{r}(t=2\text{s}) = 2\cdot 2^3\hat{x} + 2^2\hat{y} + 5\hat{z} = 16\hat{x} + 4\hat{y} + 5\hat{z}$ m

Partikelns avstånd till origo efter 2s,

$|\bar{r}(t=2\text{s})| = \sqrt{x^2+y^2+z^2} = \sqrt{16^2+4^2+5^2} = \sqrt{297} \approx 17.2$ m

b. $\bar{v} = (v_x(t), v_y(t), v_z(t)) = \dot{\bar{r}} = (\dot{x}(t), \dot{y}(t), \dot{z}(t)) = 6t^2\hat{x} + 2t\hat{y} + 0\hat{z}$ m/s

Partikelns hastighet efter 2s, $\bar{v}(t=2\text{s}) = 6\cdot 2^2\hat{x} + 2\cdot 2\hat{y} = 24\hat{x} + 4\hat{y}$ m/s

Hastighetens belopp efter 2s, $|\bar{v}(t=2\text{s})| = \sqrt{v_x^2+v_y^2+v_z^2} = \sqrt{592} \approx 24.3$ m/s

c. $\bar{a} = (a_x(t), a_y(t), a_z(t)) = \dot{\bar{v}} = (\dot{v}_x(t), \dot{v}_y(t), \dot{v}_z(t)) = \ddot{\bar{r}} = (\ddot{x}(t), \ddot{y}(t), \ddot{z}(t)) =$

$12t\hat{x} + 2\hat{y}$ m/s^2 m/s^2

Partikelns acceleration efter 2s, $\bar{a}(t=2\text{s}) = 12\cdot 2\hat{x} + 2\hat{y} = 24\hat{x} + 2\hat{y}$ m/s^2

Accelerationens belopp efter 2s, $|\bar{a}(t=2\text{s})| = \sqrt{a_x^2+a_y^2+a_z^2} = \sqrt{580} \approx 24.1$ m/s^2

Svar: **a**. $\bar{r}(t=2s) = 16\hat{x} + 4\hat{y} + 5\hat{z}$ m; $|\bar{r}(t=2s)| \approx 17.2$ m

b. $\bar{v}(t=2\text{s}) = 24\hat{x} + 4\hat{y}$ m/s ; $|\bar{v}(t=2\text{s})| \approx 24.3$ m/s

c. $\bar{a}(t=2\text{s}) = 24\hat{x} + 2\hat{y}$ m/s^2 ; $|\bar{a}(t=2\text{s})| \approx 24.1$ m/s^2

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2. **a**. $\bar{a} = \dot{\bar{v}} \Rightarrow \bar{v} = \int \bar{a}dt \Rightarrow (v_x, v_y, v_z) = (\int a_x dt, \int a_y dt, \int a_z dt)$

$\bar{a} = (24t^2, 6t, 2) \Rightarrow \bar{v} = (8t^3 + C_x, 3t^2 + C_y, 2t + C_z)$

$\bar{v}(t=0) = 0 \Rightarrow C_x = C_y = C_z = 0 \Rightarrow \bar{v} = (8t^3, 3t^2, 2t)$ m/s

$\bar{v}(t=2\text{s}) = (64, 12, 4)$ m/s

b. $\bar{v} = \dot{\bar{r}} \Rightarrow \bar{r} = \int \bar{v}dt \Rightarrow (x,y,z) = (\int v_x dt, \int v_y dt, \int v_z dt)$

$\bar{v} = (8t^3, 3t^2, 2t) \Rightarrow \bar{r} = (2t^4 + D_x, t^3 + D_y, t^2 + D_z)$

$\bar{r}(t=0) = 0 \Rightarrow D_x = D_y = D_z = 0 \Rightarrow \bar{r} = (2t^4, t^3, t^2)$ m

$\bar{r}(t=2\text{s}) = (32, 8, 4)$ m

Svar: **a**. $\bar{v}(t=2\text{s}) = (64, 12, 4)$ m/s

b. $\bar{r}(t=2\text{s}) = (32, 8, 4)$ m

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3. $\bar{a} = a_x\hat{x} = (6x+2)\hat{x}$ m/s^2 ; rörelse i en dimension $v_y = v_z = 0$

$$a_x = \dot{v}_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\cdot\frac{dx}{dt} = \frac{dv_x}{dx}\cdot v_x = \frac{d}{dx}\left(\frac{v_x^2}{2}\right)$$

$$\frac{d}{dx}\left(\frac{v_x^2}{2}\right) = 6x+2 \Rightarrow \frac{v_x^2}{2} = 3x^2 + 2x + C \Rightarrow v_x = \pm\sqrt{6x^2+4x+2C}$$

$$v_x(x=0) = 10 \Rightarrow 10 = \sqrt{2C} \Rightarrow 2C = 100 \Rightarrow v_x(x) = \sqrt{6x^2+4x+100} \text{ m/s}$$

Svar: $v_x(x) = \sqrt{6x^2+4x+100}$ m/s

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4. **a**. Partikelns läge efter 2s, $\bar{r}(t=2\text{s}) = 4\hat{r}$ m; $\theta = 2$ rad

Partikelns avstånd till origo efter 2s, $|\bar{r}(t=2\text{s})| = r = 4$ m

b. $\bar{v} = \dot{\bar{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = 2t\hat{r} + t^2 \cdot 1\hat{\theta} = 2t\hat{r} + t^2\hat{\theta}$ m/s

Partikelns hastighet efter 2s, $\bar{v}(t=2\text{s}) = 4\hat{r} + 4\hat{\theta}$ m/s; $\theta = 2$ rad

Hastighetens belopp efter 2s, $|\bar{v}(t=2\text{s})| = \sqrt{v_r^2 + v_\theta^2} = 4\sqrt{2} \approx 5.66$ m/s

c. $\bar{a} = \dot{\bar{v}} = \ddot{\bar{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} = (2 - t^2 \cdot 1)\hat{r} + (0 + 2 \cdot 2t \cdot 1)\hat{\theta} =$
$(2 - t^2)\hat{r} + 4t\hat{\theta}$ m/s^2; $\theta = 2$ rad

Partikelns acceleration efter 2s, $\bar{a}(t=2\text{s}) = -2\hat{r} + 8\hat{\theta}$ m/s^2; $\theta = 2$ rad

Accelerationens belopp efter 2s, $|\bar{a}(t=2\text{s})| = \sqrt{a_r^2 + a_\theta^2} = \sqrt{68} \approx 8.25$ m/s^2

Svar: **a**. $\bar{r}(t=2\text{s}) = 4\hat{r}$ m; $\theta = 2$ rad; $|\bar{r}(t=2\text{s})| = 4$ m

b. $\bar{v}(t=2\text{s}) = 4\hat{r} + 4\hat{\theta}$ m/s; $\theta = 2$ rad; $|\bar{v}(t=2\text{s})| \approx 5.66$ m/s

c. $\bar{a}(t=2\text{s}) = -2\hat{r} + 8\hat{\theta}$ m/s^2; $\theta = 2$ rad; $|\bar{a}(t=2\text{s})| \approx 8.25$ m/s^2

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5. Använd planpolära koordinater:

$r = \rho$ konstant; $s = \rho\theta = t^3 + 2t^2$ m $\Rightarrow \theta = \frac{1}{\rho}(t^3 + 2t^2)$ rad

$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} = -\rho \cdot \frac{1}{\rho^2}(3t^2 + 4t)^2\hat{r} + \rho \cdot \frac{1}{\rho}(6t + 4)\hat{\theta} =$$

$$-\frac{(3t^2 + 4t)^2}{\rho}\hat{r} + (6t + 4)\hat{\theta} \text{ m/s}^2$$

$$\bar{a}(t=2\text{s}) = -\frac{400}{\rho}\hat{r} + 16\hat{\theta} = 16\left(-\frac{25}{\rho}\hat{r} + \hat{\theta}\right) \text{ m/s}^2$$

$$|\bar{a}(t=2\text{s})| = 16\sqrt{\left(\frac{25}{\rho}\right)^2 + 1} = 16\sqrt{2} \Rightarrow \left(\frac{25}{\rho}\right)^2 + 1 = 2 \Rightarrow \rho = 25 \text{ m}$$

Svar: Cirkelns radie $\rho = 25$ m

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6. Båtens hastighet $\bar{v} = v_x\hat{x} + v_y\hat{y} = v_1\hat{x} + v_0(1 - (\frac{x}{b})^6)\hat{y}$

$$\hat{y}-led: \dot{y} = v_0(1 - (\frac{x}{b})^6) \Rightarrow \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dx}v_1 = v_0(1 - (\frac{x}{b})^6)$$

$$\frac{dy}{dx} = \frac{v_0}{v_1}(1 - (\frac{x}{b})^6) \Rightarrow y(x) = \frac{v_0}{v_1}\left(x - \frac{(\frac{x}{b})^7}{7\frac{1}{b}}\right) + C_y = \frac{v_0}{v_1}\left(x - \frac{b}{7}(\frac{x}{b})^7\right) + C_y$$

Sätt $y(x=0) = 0 \Rightarrow C_y = 0 \quad \Rightarrow y(x) = \frac{v_0}{v_1}\left(x - \frac{b}{7}(\frac{x}{b})^7\right)$

Sträckan blir $y(x=b) - y(x=-b) = 2y(x=b) = 2\frac{v_0}{v_1}\left(b - \frac{b}{7}(\frac{b}{b})^7\right) = \frac{12}{7}b\frac{v_0}{v_1}$

Svar: Båten hinner driva $\frac{12}{7}b\frac{v_0}{v_1}$.
